The Hachemeister's Algorithm for Heterogenous Portofolios

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ABSTRACT: *The Hachemeister's model is based on an econometric essence combined with one of numeral analysis, both applied on goods insurance. The specific feature of using this type of numeral analysis model given other types of model for premium establishment is the fact that takes in consideration both the evolution in time of contracts number and of inflation's effect over the value of demands showed during time and also the fact that it can be applied succesfully where we have to provide for an heterogenous risks portofolio.*

Keywords: heterogenous portofolios, theory of credibility, risk premiums

Suppose we have an insurance company making a contract for a real estate insurance. Normally, this contract covers different risks: fire, lightning, the coming up against, an outside object, earthquake, explosion, flood, burglary. The company has data about the reparations which the demands corresponding to each type of showing up in a certain year or certain type of contract of that repartion: normal repartion; -Poisson repartion; -classic binomial repartion.

These data and parameters of functions corresponding to each reparation were obtained from own experience also from special papers.

Normally, the pure risk premium should be calculated as an average of some aleatory variables having the reparation functions that we previous mentioned. Because the values of demands number corresponding to each type of risk are little, the available data can't be considered relevant for each type of risk. But we have information regarding a history of earthquake in Romania in the last 400 years also information regarding the frequency and evolution in time of others insured risks. With the help of these data we can calculate the parameters of reparation functions previous mentioned, for example through the method of moments' identification.

Having these data (the repartion types, the parameters of repartion function etc.) can be simulated other values for the number of demands from the last ten years, data which this time will not be real but obtained through simulation on the basis of the data that we have from a bigger data base.

Using a cycle composed by a pre-established number of simulations can be established more possible values for the next year risk premium with the help of an algorithm presented in the next pages and this premium must be paid by the insured person to the company in cause.

The respective model is based on econometric essence combined with a numeric analysis one, of course, applied as part as the insurances. The particularities of using this type of premiums are considered also the evolution in time of the number of contracts, of the inflation effect aver the value of demands showed up during time and also the fact that can be also successfully applied in cases like that one we previous presented, when we have to insure an heterogenous risks portofolio.

A series of specific features of this model for different certain situations or deliberate simplifications of reality in order to obtain an easier model to use are the Bulhman model, the Bulhman-Stramb model, the Jewell model. These models either do not consider the inflation or tackle the case easier when the certain type of contract's moments or lengths of existence in portofolio suit for each insured risk.

These models belong to a part of discipline called "The numeral analysis in actuarial mathematics", part called "The theory of credibility".

The theory of credibility is a technique of establishment of premium in insurances for different risks as part of heterogenous portofolio, technique which is based on previous experience utilization regarding the way how those insured risks have increased during the years for which we have information. This technique is used especially in the case when we have less information regarding the evolution of individual risks as part of portofolio but when we have enough information regarding evolution of the whole risks portofolio. In such cases are necessary both the scientific method's utilization and the experience to obtain in time reasonable risk premiums, both for insurer and for insured person.

Having a portofolio containing a *k* number of contracts for each contract being suit certain type of risk, it's obvious that one of them will be heterogenous: however, for a type of contract took individually are used informations regarding the risk's previous evolution respectively suited for that contract and the evolution of collective risk joined in portofolio. At the two types of experience will have attached for each risk a coefficient of credibility Z_j and $1 - Z_j$, where Z_j is the weight suited as part of insurance company's potofolio.

When $Z_j = 0$, the individual premium will be equal with the average premium on the whole portofolio: this method is used especially for almost homogenous portofolios, being less indicated their utilization as part of some portofolios which represent a certain degree of heterogeneousness.

When $Z_j = 1$, the contract is individually estimated on the basis of just the own experience regarding the demands to cover the damages suited to the respective risk. In general, this type of information regarding the individual type of risk is reduced enough, so usually this type of estimation can't be used in practice. Also, sometimes this type of estimation is inadequate (for example, in case of a risk for which never have been registered damages, the individual premium should be zero).

So, we obtain a repartion of risk on the basis of the two types of risks: individual and collective, with the shape: $\theta_j^a = Z_j\theta_j + (1 - Z_j)\bar{\theta}$.

The theory of credibility offers methods for calculus of *Z*, and also of $M(\theta_j^a)$, average that should correspond to a premium paid by the insured person.

In fact, what we are interested for are especially the moments of the degree 1 and 2 for the considered risks, either they are individual or collective, because with their help can be easy established a politics of tariff for different types of contracts. Therefore, having a risk θ and observing his producing during a period of *t* years, for the year *t*+1 the used premium will be $\mu(\theta) = M(X|\theta)$.

In general, to calculate this moment is searched solutions of linear shape:

$$\mu(\theta) = c_0 + \sum_{r=1}^{t} c_r X_r \qquad (1)$$

The coefficients are obtained through regression using the method of the most little squares, being resolved a problem of the type: [min] M $\left\{\left[\mu(\Theta) - c_0 - \sum_{r=1}^{t} c_r X_r\right]^2\right\}$.

As part of Hachemeister's model we have the next data of enter:

$$X_j = \begin{pmatrix} X_{j1} \\ . \\ . \\ . \\ X_{jt} \end{pmatrix} \qquad W_j = \begin{pmatrix} W_{1,1} & . & . & . & W_{1,t} \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ W_{t,1} & . & . & . & W_{t,t} \end{pmatrix} \qquad x = \begin{pmatrix} 1 & 1^{q_1} & . & . & . & 1^{q_{n-1}} \\ 1 & 2^{q_1} & & & & 2^{q_{n-1}} \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ 1 & t^{q_1} & . & . & . & t^{q_{n-1}} \end{pmatrix} \tag{2}$$

With Hachemeister's model help we search vectors of the type:

$$\beta(\Theta_j) = \begin{pmatrix} \beta_1 \\ \beta_2 \\ . \\ . \\ . \\ \beta_n \end{pmatrix} \tag{3}$$

These vectors model in an econometric way the evolution in time of compensation demands. With their help we obtain suited values ajustate $\hat{X}_j = \beta(\Theta_j)x$, which in essence don't have a special utility for this example. What we can do once we modeled the past evolution of compensation demands for different risks, is to extrapolate the results obtained calculating a medium expected value of demands for the next year.

$$Y_j = \beta(\Theta_j)\begin{pmatrix} 1 \\ (t+1)^{q_1} \\ \vdots \\ (t+1)^{q_{n-1}} \end{pmatrix} \ldots j = 1,k \tag{4}$$

With this value help it can be established a premium for these risks that consider all the aspects we previous mentioned. These values will be:

For the year t+2 it can be used again the formula:

$$Y_j = \beta(\Theta_j)\begin{pmatrix} 1 \\ (t+2)^{q_1} \\ \vdots \\ (t+2)^{q_{n-1}} \end{pmatrix} \ldots j = 1,k \tag{5}$$

Or can calculate again $\beta(j)$ with the help of data from the years l, *t*+l so, the data's importance to be bigger (also through the introducing of the observations made in the last year, the most significant one, but also through the growth of the period of time took in consideration).

The simulation is used to obtain an average value of these Y_j after many cycles of simulation. As we presented before, this value is much more appropriated by the real average (which can be calculated through analytic methods, situation that stimulates the simulation) than any other individual value obtained after a single cycle of simulation.

Further on we'll present the algorithm built on basis of Hachemeister's model, algorithm that is based on the interactive calculus of credibility coefficients Z_j, until is obtained a value near a satisfying estimation of value of *A*; after that is calculated $\beta(j)$ and then Y_j.

The algorithm based on Hachemeister's method is the next one:

Read k,t,n, epsilon
Read Wj; j=l,k
Read x;
Read Xj;
A:=In;

$$\beta j := (x' W_j^{-1} x)^{-1} x' W_j^{-1} X_j ;$$

Repeat

A1:=A;

$$Z_j := A(A + s^2 (x' W_j^{-1} x)^{-1})^{-1} ;\ for\_j = l,k$$

$$Z := \sum_{j=1}^{k} Z_j ;$$

$$b := Z^{-1} \sum_{j=1}^{k} Z_j \beta_j ;$$

$$s^2 = \frac{1}{k(t-n)} \sum_{j=1}^{k} (X_j - x\beta_j)' W_j^{-1} (X_j - x\beta_j) ;$$

$$A = \frac{1}{k-1} \sum_{j=1}^{k} Z_j (\beta_j - b)(\beta_j - b)'$$

Until ($|A1 - A| <$ *epsilon);*

$$\beta(j) := Z_j \beta_j + (I_n - Z_j) b ;$$

$$Y(j) := \beta(j) x .$$

All the elements that we presented in this algorithm are matrix.

Of course, this algorithm is part of a simulation cycle for different values of aleatory vectors X_j (j = 1,k) to obtain an average value *Y*, that can be used to establish the premiums for the next year.

The problem that can appear is that not always the algorithm leads us to a solution of equilibrium. As in other trajectory study models case of other cybernetic systems it's possible that starting from an initial situation to enter in a stable equilibrium cycle of 2,3,4 etc degree.

In this case, in a real situation it should be calculated analytically which of the obtained values realizes the minimum as part of the most little squares method that is the theoretical base of the algorithm. In this program we'll introduce as a finite condition of algorithm also the touching of a number with 150 iterations, number that's considered sufficient to touch the equilibrium value. If it's passed over the iteration 150 without being obtained the equilibrium value, is registered the next generation of enter data, X_j, not being recorded the simulation cycle number made.

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